# Modeling the Flight of a TOW Missile

Congratulations on graduating from IOBC and being assigned to $2^{nd}/325^{th}$ Airborne Infantry Regiment, (renowned for its mathematicians as well as its combat jumps) at Fort Bragg, North Carolina. Still sweaty from PT, your CO catches up with you and hands you the following warning order. On Wed, 6 Oct you will be dropped into Weinstein Republic vicinity ___a____. In this project we model this point as ___b____. Your platoon's mission is to establish a battle position to provide coverage of TRP AB001 and knock out the lead vehicle escorting any supply column that may move into sector across the bridge in the 2658 grid square. This will assist the joint attack and destruction of the column. You must provide the Battalion Commander a written report on Tue, 5 Oct, addressing the following missions.

## Mission One - Battle Position Analysis

http://www.redstone.army.mil/history/systems/TOW.html

**a)** Primary Position. Determine an eight digit grid for your TOW position and explain your rationale. If the TOW were to travel along a straight path from your position to the bridge, model the path of the TOW in 3-D using parametric equations. Explain why this path is or is not realistic.

**b)** Alternate Position. Your PSG (who earned a Ph.D. in Operations Research from MIT) offers 245592 as an alternate position for the TOW team. He models a path for the flight of the TOW fired from the alternate position in the *xy*-plane as $y(x) = 0.0625x^2 - 3.3625x + 104.03$, where $x$ is measured in kilometers to the East and $y$ is measured in kilometers to the North; the origin is located at grid coordinate 000000. Finally, he models the elevation of the TOW along this path as $z(x) = (0.6438x^2 - 63.09x + 1244.36)\frac{1}{1000}$, where $z(x)$ is kilometers above the river elevation and $x$ is kilometers to the East. Separately plot these equations in 2-D. Then, determine and graph the 3-D set of parametric equations that models the trajectory. Use $x(t) = 24.5 + \frac{t}{2}$ ($t$ is in seconds) in your parametrization, so that when $t = 0$, the TOW will be at this position. Do your 2-D and 3-D graphs depict realistic paths which cross the bridge? Finally, you find out from the experienced NCOs in your platoon that a TOW usually travels its maximum effective range in about 8 seconds. Using your model, determine the speed of the TOW in the impact zone and compare this speed to that which you learned from your NCOs. Does your speed seem reasonable? If not, explain how you can modify your PSG's model in order to make it more realistic.

**c)** Supplementary Position. Your first SL (who earned a MS in Applied Mathematics from Brown University) offers 260588 as a supplementary position for the TOW team. He models a path for the flight of the TOW fired from the supplementary position in the *xy*-

plane as $y(x) = 0.0125x^2 - 0.65x + 67.25$, where $x$ is again measured in kilometers to the East and $y$ is measured in kilometers to the North. Finally, he models the elevation for the TOW along this path as

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$z(x) = (1.024x^2 - 106.5x + 2136.7)\dfrac{1}{1000}$, where *z(x)* is kilometers above the river elevation and $x$ is kilometers to the East. Conduct the same analysis as in part b) and use $x(t) = 26.0 + t$ (*t* is in seconds) in your parameterization so that when *t = 0*, the TOW will be at this position.

**Mission Two - Terrain Analysis**

**a)** Terrain & Vegetation. Research of the vegetation in your area of operations tells you that most trees reach a maximum height of ___c___ meters in this region. Determine from your map reconnaissance if the vegetation or terrain will prevent mission success from any of your three positions. If there is an obstruction, is it possible that you could still use the position?

**b)** Recommendation. Based on your analysis, which position do you recommend your TOW team occupy to accomplish this mission? Why?

**Mission Three - Safety**. As a final thought, let's examine the safety issue for the citizens of Weinacht. You know from IOBC that if the line on the TOW breaks, it usually occurs at the first half second of flight. If the missile does detach from the line, the path of the missile is very erratic and can be modeled by the following parametric equations:

$$a(s) = s, \qquad b(s) = k\left[\sin\left[\frac{p}{2}(3s-1)\right]+1\right], \quad \text{and} \quad c(s) = -0.6\left[\frac{s^4}{4} - \frac{5s^3}{3} + \frac{7s^2}{2} - 3s\right]\frac{1}{1000}$$

where *s* is in seconds, *a(s)* is the distance in kilometers to the East from the point where the TOW wire breaks; *b(s)* is the distance in kilometers to the North from the point where the TOW wire breaks; and *c(s)* is the height in kilometers from the point where the TOW wire breaks (i.e. a new origin where *s = 0*). Assume that the TOW will run out of fuel after it has traveled ___d___ meters. Once the TOW runs out of fuel, it becomes a free falling object. Your commander (a literature standout at USMA) would like to know if the TOW does detach from the wire, will it clear the town of Weinacht? Using your recommended TOW position, determine whether or not a "loose" TOW round will land in the town of Weinacht.