Interdisciplinary Lively Applications Project

Modeling Production

"Production of the Bangalore Torpedo"

Introduction.
In a profit maximizing industry, economists organize decisions that firms can make into three classes: 1) how best to employ existing plant and equipment – the short run; 2) what new plant and equipment to select – the long term; and 3) how to encourage the development of new technology - the very long term. In the short run, the only input into production that is considered to be adjustable is the number of laborers. In the long term, the amount of plant and equipment can be adjusted. Finally, in the very long run the technological possibilities available to the company can be changed. In this project, you will be examining a certain company where the number of laborers can be varied in the short run and the number of operating plants can be varied in the long run[1].

Mission 1 (Production in the Short Term)

Your company titled **BANGALORES R US** produces bangalore torpedoes (commonly called bangalores), a demolition used for breaching wire obstacles. It produces these devices for the Army Corps of Engineers according to the production function $Q = 12L + 29L^2 - 1.1L^3$, where Q is the number of bangalores produced per year and L is the number of laborers used per year for production. Additionally, the price of labor and other consumable materials (P_L) is \$7000 per laborer year. Each bangalore sells for \$125.

a) Is there a range of values of production for the bangalores that would be profitable? If so, what is this range? Show this graphically.
b) What is the maximum profit that can be obtained by producing the bangalores? Show why this is a maximum using a graphical analysis of the first and second derivatives.
c) Economists refer to marginal revenue (*MR*) as the change in total revenue attributable to a one-unit change in output. The marginal cost (*MC*) is the change in total cost attributable to a one-unit change in output. In other words, marginal revenue is the derivative of the total revenue function and marginal cost is the derivative of the total cost function. Using your work above, explain why economists use the equation $MR = MC$ to maximize profits.
d) "The law of diminishing returns states that if increasing amounts of a variable factor (in this case – laborers) are applied to a production line, eventually a situation will be reached in which each additional unit of the variable factor adds less to the total product than the previous unit[2]." Graph the instantaneous rate of change of the quantity produced per year with respect to the number of laborers. From this graph,

explain where the point of diminishing returns for the laborers is and why. What is a physical explanation in your company for the point of diminishing returns?

Mission 2 (Production in the Long Term– Revisiting Discrete Dynamical Systems)

After keeping close eye on the production for the last few months, you have been able to collect the following discrete data to help your analysis for the long term. Each cell represents a given number of bangalores produced per year with a combination of laborers and the units of capital (the number of plants in operation). Armed with this data, you are prepared to predict the amount of output for varying combinations of the number of laborers and plants.

# of Laborers

# of Plants	10	20	30	40	50	60	70	80	90	100	110	120
1	21	42	63	84	105	126	147	168	189	210	231	252
2	44	88	132	176	221	265	309	353	397	441	485	529
3	93	185	278	370	463	556	648	741	833	926	1019	1111
4	194	389	583	778	972	1167	1361	1556	1750	1945	2139	2334
5	408	817	1225	1634	2042	2450	2859	3267	3676	4084	4493	4901
6	858	1715	2573	3431	4288	5146	6004	6861	7719	8577	9434	10292
7	1801	3602	5403	7204	9005	10807	12608	14409	16210	18011	19812	21613
8	3782	7565	11347	15129	18911	22694	26476	30258	34041	37823	41605	45387

a) Plot this discrete multivariable function in 3-D.
b) Given that you have *5* plants, how could you model the data discretely so that you could predict your output. Predict the output for 190 laborers.
c) Is a discrete analysis a good approach for this company? Why or why not?

Mission 3 (Production in the Long Term– A Continuous Analysis)

An isoquant curve is a level curve that shows all technologically efficient factor combinations for producing a specified amount of output. In your situation, an isoquant curve would be any level curve which has all possible combinations of number of laborers and plants for a specified production quantity. Similarly, an isocost curve is a level curve that shows alternative combinations of factors that a company can buy for the same cost. For your company, the isocost curve would be any level curve which has all possible combinations of number of laborers and plants for a specified production cost.

Your trusty mathematician and Vice President Randy Donaldson, is a strong advocate of modeling production continuously. He determined the following relationship: $Q = f(L,K) = (6K-K^2)(14L-L^2)$, for $K < 6$ and $L < 14$. Assume that the total cost (TC) is of the form $TC = P_L*L + P_K*K$, where P_L is the price of ten laborers per year, P_K is the cost of operating a plant per year, L is the number of laborers used per year, and K is the number of plants used per year. In both of these models, L is in tens.

a) Find and classify all extrema for the production line. Is there a combination of capital and laborers that will give a maximum output? If so, what is it?
b) Given that you have 3 plants in operation, what is the maximum amount you can produce?
c) Explain using a graph of Q what an isoquant would be in the KL plane.
d) Given that the costs are 7000 dollars / plant-year and 15000 dollars / 10 laborers-year, (P_K) and (P_L) respectively, what is the slope of the isocost line? How about in terms of P_K and P_L? For this P_K and P_L, you can spend only $63,940 dollars on production per year. What is the maximum output you can obtain?
e) In general, would you use a discrete or continuous analysis for any company that produces items? Explain.

Mission 4 (Production in the Long Term Using Level Curves)

In 1928, Charles Cobb and Paul Douglas modeled the growth of the American economy [3]. Their model, $P(L,K) = AL^a K^b$, later to become known as the famous Cobb-Douglas model, predicted the production output in thousands, based on the amount of capital and the number of laborers used. In this equation, A, a, and b are constants.

Suppose that using this model, you would like to investigate the optimal mix of laborers and capital for 4000 bangalores. You know that the price of capital and labor per year is $10,000 and $7,000 respectively, and that the constants in the Cobb-Douglas model for your company are $A = 1.2$, $a = .3$, and $b = .6$. Your total cost equation is still of the form $TC = P_L*L + P_K*K$, the units of your production function is thousands of bangalores, while the units on L in both functions is in tens.

a) Given that the costs are $10,000 / plant-year and $7,000 / 10 laborers-year, (P_K) and (P_L) respectively, graph the level curve of the production function and the contraints for total annual cost given several budget possibilities of $63,940, $55,060, and $71,510. (Plot labor on the vertical axis and capital on the horizontal axis).
b) Identify from this graph which of the three budgets you should strive to meet. Approximate the optimal mix of labor and capital for the desired budget.
c) Using Lagrange Multipliers, explain why you picked the isocost curve and optimal mix that you did.

Economist use the following relationship to determine if the production costs are minimized in the long run: $\frac{MP_K}{p_K} = \frac{MP_L}{p_L}$

where MP_K is the marginal productivity of capital, or more simply the partial derivative of Q with respect to K. Similarly, MP_L is the marginal productivity of labor. Whenever the two sides of the above equation are not equal, there are possibilities for increasing or decreasing the number of laborers or units of capital in order to reduce costs.

d) Given that $Q = f(K,L)$, K and L are functions of other variables, one of which is management sentiment (S). Using the chain rule, determine the general form of the instantaneous rate of change of the quantity produced with respect to management sentiment.

e) The differential, dQ, or the change in the quantity produced, would be $dQ = \frac{\delta Q}{\delta K} dK + \frac{\delta Q}{\delta L} dL$. Along an isoquant curve, $dQ = 0$. Using this differential, derive the popular economics equation $\frac{MP_K}{p_K} = \frac{MP_L}{p_L}$.

f) In the very long run, technology changes have an impact on production. What impact on the quantity produced do you think technology advances have? Describe what happens to the isoquant curves as technology advances.

NOTES

[1]Richard G. Lipsey and Paul N. Courant, *Economics*. (New York, New York: HarperCollins Publishers Inc, 1996),162.

[2]Richard G. Lipsey and Paul N. Courant, *Economics*. (New York, New York: HarperCollins Publishers Inc, 1996),165.

[3]James Stewart, *Calculus Concepts and Contexts*. (Pacific Grove, CA: Brooks/Cole Publishing Company, 1998),749.